

Problem 2.86

[Difficulty: 3]

2.86 In a food industry process, carbon tetrachloride at 20°C flows through a tapered nozzle from an inlet diameter $D_{in} = 50$ mm to an outlet diameter of D_{out} . The area varies linearly with distance along the nozzle, and the exit area is one-fifth of the inlet area; the nozzle length is 250 mm. The flow rate is $Q = 2$ L/min. It is important for the process that the flow exits the nozzle as a turbulent flow. Does it? If so, at what point along the nozzle does the flow become turbulent?

Given: Geometry of and flow rate through tapered nozzle

Find: At which point becomes turbulent

Solution:

Basic equation For pipe flow (Section 2-6) $Re = \frac{\rho \cdot V \cdot D}{\mu} = 2300$ for transition to turbulence

Also flow rate Q is given by $Q = \frac{\pi \cdot D^2}{4} \cdot V$

We can combine these equations and eliminate V to obtain an expression for Re in terms of D and Q

$$Re = \frac{\rho \cdot V \cdot D}{\mu} = \frac{\rho \cdot D}{\mu} \cdot \frac{4 \cdot Q}{\pi \cdot D^2} = \frac{4 \cdot Q \cdot \rho}{\pi \cdot \mu \cdot D} \quad Re = \frac{4 \cdot Q \cdot \rho}{\pi \cdot \mu \cdot D}$$

For a given flow rate Q , as the diameter is reduced the Reynolds number increases (due to the velocity increasing with A^{-1} or D^{-2}).

Hence for turbulence ($Re = 2300$), solving for D $D = \frac{4 \cdot Q \cdot \rho}{2300 \cdot \pi \cdot \mu}$

The nozzle is tapered: $D_{in} = 50$ mm $D_{out} = \frac{D_{in}}{\sqrt{5}}$ $D_{out} = 22.4$ mm

Carbon tetrachloride: $\mu_{CT} = 10^{-3} \cdot \frac{N \cdot s}{m^2}$ (Fig A.2) For water $\rho = 1000 \cdot \frac{kg}{m^3}$

$SG = 1.595$ (Table A.2) $\rho_{CT} = SG \cdot \rho$ $\rho_{CT} = 1595 \cdot \frac{kg}{m^3}$

For the given flow rate $Q = 2 \cdot \frac{L}{min}$ $\frac{4 \cdot Q \cdot \rho_{CT}}{\pi \cdot \mu_{CT} \cdot D_{in}} = 1354$ LAMINAR $\frac{4 \cdot Q \cdot \rho_{CT}}{\pi \cdot \mu_{CT} \cdot D_{out}} = 3027$ TURBULENT

For the diameter at which we reach turbulence $D = \frac{4 \cdot Q \cdot \rho_{CT}}{2300 \cdot \pi \cdot \mu_{CT}}$ $D = 29.4$ mm

But $L = 250$ mm and linear ratios leads to the distance from D_{in} at which $D = 29.4$ mm $\frac{L_{turb}}{L} = \frac{D - D_{in}}{D_{out} - D_{in}}$

$$L_{turb} = L \cdot \frac{D - D_{in}}{D_{out} - D_{in}} \quad L_{turb} = 186 \text{ mm}$$